# *On the Complete System of the "Grundformen" of the Binary Quantic of the Ninth Order.*

By J. J. Sylvester.

## Enumeration of the Irreducible Invariants and Covariants of the Binary Quantic of the Ninth Order.

| Degree in the Coefficients. | Order in the Variables. | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 21 | 22 |
| 1 | | | | | | | | | | 1 | | | | | | | | | | | |
| 2 | | | 1 | | | | 1 | | | | 1 | | | | 1 | | | | | | |
| 3 | | | | 1 | | 1 | | 1 | | 2 | | 1 | | 1 | | 1 | | 1 | | 1 | |
| 4 | 2 | | | | 2 | | 2 | | 3 | | 2 | | 2 | | 2 | | 1 | | 1 | | 1 |
| 5 | | 1 | | 3 | | 4 | | 4 | | 3 | | 4 | | 2 | | 2 | | | | | |
| 6 | | | 4 | | 4 | | 6 | | 6 | | 3 | | 3 | | | | | | | | |
| 7 | | 4 | | 7 | | 8 | | 7 | | 5 | | | | | | | | | | | |
| 8 | 5 | | 8 | | 10 | | 10 | | 2 | | | | | | | | | | | | |
| 9 | | 9 | | 14 | | 10 | | 2 | | | | | | | | | | | | | |
| 10 | 5 | | 15 | | 14 | | | | | | | | | | | | | | | | |
| 11 | | 17 | | 16 | | | | | | | | | | | | | | | | | |
| 12 | 14 | | 23 | | | | | | | | | | | | | | | | | | |
| 13 | | 25 | | | | | | | | | | | | | | | | | | | |
| 14 | 17 | | 9 | | | | | | | | | | | | | | | | | | |
| 15 | | 26 | | | | | | | | | | | | | | | | | | | |
| 16 | 21 | | | | | | | | | | | | | | | | | | | | |
| 17 | | 5 | | | | | | | | | | | | | | | | | | | |
| 18 | 25 | | | | | | | | | | | | | | | | | | | | |

The foregoing table has been calculated, out of the funds voted by the British Association, under my superintendence, by Mr. Franklin, Fellow of Johns Hopkins University. A statement of the method employed will be given in a future number of the *Journal.*

The total number of irreducible forms will be seen from the table to be 415. The highest degree in the coefficients is 18, and the highest order in

the variables 22. The *representative* generating function in this case (as in all others which have been hitherto treated, with the sole exception of the seventhic) has a *finite* numerator.

The total number of groundforms for the orders 0, 2, 4, 6 respectively (counting, as one ought to do, the absolute constant as one of them) is 1, 3, 6, 27, which becomes a regular series on increasing 6, which corresponds to a square index 4, in the proportion of 2 : 3. In like manner, for the orders 1, 3, 5, 7, 9, the series is 2, 5, 24, 125, 416, which, on increasing the last term corresponding to the square index 9 in the ratio 2 : 3, forms an almost regular progression 2, 5, 24, 125, 624, highly suggestive of the geometrical series 1, 5, 25, 125, 625. It seems then to be a not altogether improbable conjecture, that the number of groundforms for 10, which I hope very soon to get completely worked out, will be in the neighbourhood of a ratio of equality to 243,* and for 11, which there is not much prospect of calculating for some time to come, a number not very far out from a ratio of equality to 3125. In the next, or next but one, number of the *Journal* I hope to set out a synoptical table of the groundforms for all orders up to 10 inclusive, with their reduced and representative generating functions, as also for combinations of the orders: 2, 3; 2, 4; 3, 3; 3, 4; 4, 4; all the materials for which, with the exception of what pertains to the covariants *proper* of the tenthic, are already in existence.

## Extract of a Letter from Sig. A. de Gasparis to Mr. Sylvester.

. . . J'ai trouvé certaines séries dans lesquelles les éléments tels que le rayon vecteur, les anomalies excentriques et vraies, etc., sont exprimés en fonction de l'anomalie moyenne *donnée en parties du rayon* sans sinus ou cosinus. Comme essai, je vous comunique les suivantes dans lesquelles $e =$ excentricité, $v$ et $M$ anomalie vraie et moyenne. En outre $a =$ demigrand axe, $i$, $\phi$ inclinaison et noend, $\pi$ perihelie, $\psi = \pi - \phi$. J'ai trouvé

$$v = \sqrt{\frac{1+e}{1-e}}\left\{\frac{M}{1-e} - \frac{M^3}{6}\,\frac{2e}{(1-e)^4} + \frac{M^5}{120}\,\frac{2e+2ae^2}{(1-e)^7} - \frac{M^7}{5040}\,\frac{18e+22e^2-900e^3}{(1-e)^{10}} + \dots\right\};$$

et posant

$$H = (1-e)\sin\psi + \frac{M}{1}\sqrt{\frac{1+e}{1-e}}\cos\psi - \frac{M^2}{2}\,\frac{\sin\psi}{(1-e)^2} - \frac{M^3}{6}\sqrt{\frac{1+e}{1-e}}\,\frac{\cos\psi}{(1-e)^3}$$

*The number of groundforms for the Octavic (I quote from memory) is 70, not more inferior to 81 than might have been anticipated, when the composite form of the number 8 is taken into account. It seems likely that for 10, 243 is at all events a superior limit.